Boolean Algebra, Axioms and Rules

$\mathcal{B}(+, \cdot, \neg)$ is a Boolean Algebra, if axioms I) through V) hold.

1 Axioms

I) Associativity: $a + (b + c) = (a + b) + c$, likewise for $\cdot$.

II) Commutativity: $a + b = b + a$, likewise for $\cdot$.

III) Distributivity: $a + bc = (a + b)(a + c)$, $a(b + c) = ab + ac$.

IV) Absorption: $a + ab = a$, $a(a + b) = a$.

V) Inverse/Complements: $0, 1 \in \mathcal{B}$ and $a + \overline{a} = 1$, $a\overline{a} = 0$,

2 Rules

1. $a + 0 = a$
2. $a \cdot 1 = a$
3. $a \cdot a = a$
4. $a + a = a$
5. $a + 1 = 1$
6. $a \cdot 0 = 0$
7. $\overline{0} = 1$
8. $\overline{1} = 0$
9. uniqueness of the inverse $ax = 0$ and $a + x = 1 \Longrightarrow x = \overline{a}$
10. $\overline{\overline{a}} = a$
11. $\overline{ab} = \overline{a} + \overline{b}$ (DeMorgan)
12. $\overline{a + b} = \overline{a} \cdot \overline{b}$ (DeMorgan)
13. $a + \overline{a}b = a + b$

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