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Handout – Definitions

1 Set With One Operation

Let G be a set with $|G| \geq 1$.

A *binary operation* $\circ$ on G is a map

$$\circ : G \times G \longrightarrow G.$$

2 Semigroup

$(G, \circ)$ is called a *semigroup* if $\circ$ is associative:

$$(a \circ b) \circ c = a \circ (b \circ c) \quad \forall a, b, c \in G.$$

3 Monoid

A semigroup $(G, \circ)$ is called a *monoid* if there exists a neutral element e with

$$a \circ e = e \circ a = a \quad \forall a \in G.$$

4 Group

A monoid $(G, \circ)$ is called a *group* if, for each $a \in G$, there exists an inverse element a^{-1} with

$$a^{-1} \circ a = e.$$

5 Abelian Group

A group $(G, \circ)$ is called an *abelian group* or *commutative group* if

$$a \circ b = b \circ a \quad \forall a, b \in G.$$

6 Ring: Two Operations

Let R be a set with $|R| \geq 1$.

Let „+" be an operation which makes $(R, +)$ an abelian group. The neutral element of R is called 0.

Let „·" be an operation which makes $(R, \cdot)$ an semigroup.

Then R is called a *ring* if $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$ for all $a, b, c \in R$.

The ring R is called *commutative* if $ab = ba$ for all $a, b \in R$.

The ring R is called *ring with identity* if $1 \in R$ with $a \cdot 1 = 1 \cdot a = a$ for all $a \in R$.

7 Field: Two Operations

Let F be a set with $\{0, 1\} \in F$ and $0 \neq 1$.

Let F be a commutative ring with identity.

Let $(F \setminus \{0\}, \cdot)$ be an abelian group.

Then F is called a *field*.

The smallest number $m \in \mathbf{N}$ such that

$$\underbrace{1 + 1 + 1 + \ldots + 1}_{m} = 0$$

is called the *characteristic* of the field.