

Boolean Algebra, Axioms and Rules

1 Axioms

- I) Associativity: $a + (b + c) = (a + b) + c$, likewise for \cdot .
- II) Commutativity: $a + b = b + a$, likewise for \cdot .
- III) Distributivity: $a + bc = (a + b)(a + c)$, $a(b + c) = ab + ac$.
- IV) Absorption: $a + ab = a$, $a(a + b) = a$.
- V) Inverse/Complements: $a + \bar{a} = 1$, $a\bar{a} = 0$,

2 Rules

1. $a + 0 = a$
2. $a \cdot 1 = a$
3. $a \cdot a = a$
4. $a + a = a$
5. $a + 1 = 1$
6. $a \cdot 0 = 0$
7. $\bar{0} = 1$
8. $\bar{1} = 0$
9. uniqueness of the inverse $ax = 0$ and $a + x = 1 \implies x = \bar{a}$
10. $\bar{\bar{a}} = a$
11. $\overline{ab} = \bar{a} + \bar{b}$ (DeMorgan)
12. $\overline{a + b} = \bar{a} \cdot \bar{b}$ (DeMorgan)
13. $a + \bar{a}b = a + b$